Logistic Modeling
Guided notes

Warm up:

Watch the video on exponential growth and logistic modeling.

a. What is the algebraic form for logistic modeling?

$f(t) = \frac{c}{1+Be^{-kt}}$

b. What is the carrying capacity?

maximum value in the problem
ex. max population

c. What does the critical value represent?

the change from exponential growth

d. How do you find the initial value for a logistic function?

Sub. 0 into the function and evaluate. It is the y-intercept (0, y)

Basic Skills Review:

a. Given the following function find $f(3)$

$f(t) = 3e^{-.01t}$

$f(3) = 3e^{-.01(3)}$

$f(3) = 2.9113$

b. Given the following function find f(0). What is the name of this coordinate?

$f(t) = \frac{3}{2+5e^{-.12t}}$

$f(0) = \frac{3}{2+5e^{-.12(0)}} = \frac{3}{2+5e^{0}} = \frac{3}{2+5(1)} = \frac{3}{7}$

$(0, 3/7)$ → y-intercept

c. Solve the following exponential equation algebraically then check using a graphing calculator.

$\frac{7}{10} = \frac{10e^{-.12t}}{10}$

$.7 = e^{-.12t}$

$\ln(.7) = \ln e^{-.12t}$

$\frac{\ln(.7)}{-.12} = \frac{-.12t \ln e}{-.12}$

$t = 2.9723$

graphing calc:

$y_1 = 7$

$y_2 = 10e^{-.12t}$ > intersect → (2.9723, 7) ✓

$t = 2.9723$

Logistic Function: $f(t) = \frac{c}{1+Be^{-kt}}$

c: carrying capacity

k: rate

$\frac{c}{1+B}$: starting value

The fields that would use logistic growth are, but not limited to; biology, ecology, sociology, agriculture, economics (Baugh, 2025), engineering, and chemistry.

Logistic Modeling
Guided notes

Example 1:

A contagious canine cough spreads through a kennel. The number of infected dogs, $N(t)$, at time t (in days) can be modeled by the logistic equation:

$$N(t) = \frac{75}{1 + 36.5e^{-0.5t}}$$

a. What is the limiting value (carrying capacity) of infected dogs in this kennel?
a) 2 dogs b) 10 dogs c) 75 dogs d) 36.5 dogs

b. How many dogs were initially infected when the outbreak began (at $t = 0$)?
a) 1 dog b) 2 dogs c) 3 dogs d) 10 dogs

$\frac{75}{1+36.5e^{-.5(0)}} = \frac{75}{1+36.5(1)}$

c. Approximately how many dogs will be infected after 3 days?
a) 2 dogs b) 6 dogs c) 8 dogs d) 15 dogs

$\frac{75}{1+36.5e^{-.5(3)}} = 8.2019$

Example 1A: You Try

The logistic growth function $f(t) = \frac{117{,}000}{1+4{,}700e^{-t}}$ describes the number of people, $f(t)$, who have become ill with influenza (the flu) t weeks after its initial outbreak in a particular community.

A) How many people became ill with the flu when the epidemic began?

$\frac{117{,}000}{1+4700e^{-0}} = 2.489 \approx 2$ people

B) How many people had the flu after 4 weeks?

$\frac{117000}{1+4700e^{-4}} = 134.35 = 134$ people were infected after 4 weeks

C) What is the limiting size of the population?

117,000; the entire population

D) In how many weeks will the number of people with the flu reach 7,000?

$(1+4700e^{-t})\ 7000 = \frac{117000}{1+4700e^{-t}} \cdot (1+4700e^{-t})$

$\frac{7000(1+4700e^{-t})}{7000} = \frac{11700}{7000}$

$1 + 4700e^{-t} = \frac{117}{70}$

$\frac{4700e^{-t}}{4700} = \frac{47}{70} - \frac{70}{4700}$

$e^{-t} = .0014286$

$\ln e^{-t} = \ln(.0014286)$

$\frac{-t \ln e}{-1} = \frac{-8.85}{-1} \rightarrow t = 8.85$ weeks

Logistic Modeling
Guided notes

Example 2:

A patch of mold in a humid bathroom initially covers 5 square centimeters. The maximum area it can spread to on the wall is 200 square centimeters. If, after 48 hours, the mold has spread to 20 square centimeters, find the logistic equation describing its growth.

Parameters:
$C = 200 \rightarrow$ max capacity
$B = 39$
$k = 0.0305$

$f(x) = \frac{200}{1+39e^{-0.0305x}}$; x is in hours

B: $5 = \frac{200}{1+Be^{-k(0)}}$

$(1+B)\ 5 = \frac{200}{1+B} \cdot (1+B)$

$\frac{5(1+B)}{5} = \frac{200}{5}$

$1+B = 40 \rightarrow B = 39$

k: $20 = \frac{200}{1+39e^{-k(48)}}$

$\frac{20}{20}(1+39e^{-k(48)}) = \frac{200}{20}$

$1+39e^{-k(48)} = 10$

$\frac{39e^{-k(48)}}{39} = \frac{9}{39}$

$e^{-k(48)} = \frac{9}{39}$

$\ln e^{-k(48)} = \ln\left(\frac{9}{39}\right)$

$\frac{-k(48)\ln e}{-48} = \frac{\ln\left(\frac{9}{39}\right)}{-48}$

$k = 0.0305$

Example 2A: You Try

A remote island community has a total population of 800 people. When the flu first arrives, 5 individuals are infected. After 7 days, 100 people are infected. Assuming the flu spread follows a logistic model, determine the equation that models the number of infected individuals over time.

a. What is the carrying capacity (c) for the spread of this strain of the flu?

800

b. What is the initial number of infected people $N(0)$?

5

c. Using the general form of the logistic function $N(t) = \frac{c}{1+Be^{-kt}}$, where $N(0) = \frac{c}{1+B}$, use the given information to find the value of B.

$(1+B)\ 5 = \frac{800}{1+B} \cdot (1+B)$

$\frac{5(1+B)}{5} = \frac{800}{5} \rightarrow 1+B = 160 \rightarrow B = 159$

d. Use the data point ($t = 7$ days, $N = 100$ people) to find the growth rate k. Show your work.

$100 = \frac{800}{1+159e^{-k(7)}}$

$\frac{100(1+159e^{-7k})}{100} = \frac{800}{100}$

$1+159e^{-7k} = 8$

$\frac{159e^{-7k}}{159} = \frac{7}{159}$

$e^{-7k} = \frac{7}{159}$

$\frac{-7k\ln e}{-7} = \frac{\ln\left(\frac{7}{159}\right)}{-7} \rightarrow k = 0.4461$

e. Write the complete logistic equation that models the spread of the flu in the island community.

$f(t) = \frac{800}{1+159e^{-0.4461t}}$

f. Predict how many people will be infected with the flu 2 weeks, according to your model.

$\rightarrow 14$ days

$f(14) = \frac{800}{1+159e^{-.4461(14)}}$

$= 611.45 \rightarrow$ after 2 weeks 611 people will be infected with the flu.